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**PERT AND LOB SIMPLIFIED INTEGRATED MODEL FOR DETERMINING
PROBABILISTIC SCHEDULING OF REPETITIVE PROJECTS**

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ABSTRACT

In a construction project environment there is always a risk or chance that adverse conditions could occur, which may cause the project to fail, or couldn't meet its planned objectives. The importance of planning and time scheduling has emerged in the last decade and it becomes an essential step for any construction project to reduce this risk. The project schedule should reflect all of the work associated with delivering the project on time. Without a full and complete schedule, the project manager will be unable to communicate the complete effort, in terms of cost and resources, necessary to deliver the project. Most current methods of time schedules and estimating have been converted to software programs in order to be easy to be used by planners and to avoid manual mistakes. Presently, experts in the fields of time planning are searching for better software they can depend on, that is more powerful, easier and serves different kinds of projects. This paper develops software that is essential for improving time planning and estimating for projects that repeat linearly and focuses on features of PERT and LOB for calculating expected completion probability of any repetitive construction project within a specified end date. The software is designed by EXCEL spreadsheet system to provide the factors that affect projects end date probability. At the end, illustrative cases of study are presented to validate and verify the proposed software.

KEYWORDS: Repetitive Construction Projects; Construction management; Program Evaluation and Review Technique; Line of Balance; Excel Programming; Software.

INTRODUCTION

This research will focus on one of the constrains that control any construction project which is time. Preparing time schedule for any project is one of the most major steps to start the project and in sometimes may cause to cancel the project or delay its start time. Furthermore, time estimates are important as inputs into other techniques used to organize and structure all projects. Without an accurate estimate, no project can be completed within the budget and the target completion date. This research will concern with two methods of time estimating; Line of Balance (LOB) and Program Evaluation & Review Technique (PERT). It introduces an EXCEL program in order to apply PERT method on projects that consist of repetitive activities. By combining the two methods together it will be easy to find out the probabilities of finishing repetitive projects in certain date. Also this combination help to achieve an efficient and sustainable time schedules prediction. This program is the first step to study the effect of the repetition on the probability of finishing the final project in certain time. Results acquired from this research can be used by engineers and academics to predict the probability of finishing repetitive linear projects in certain dates.

RESEARCH OBJECTIVES

The purpose of this study is to (1) Prepare EXCEL program to calculate early and late dates using Line of Balance (LOB) method; (2) Prepare EXCEL program that calculate the probability of finishing projects, which is solved by using Line of Balance (LOB) method, in certain date using Project Evaluation and Review Technique (PERT) method. (3) Find out factors that affect the end date probability for the projects that repeat linearly.

LITERATURE REVIEW

PERT (project evaluation and review technique) was first developed during the 1950s through the efforts of the U.S. The Navy used PERT to coordinate the efforts of some 3,000 contractors involved with the project. Experts credited

PERT with shortening the project duration by two years. Since then, all government contractors have been required to use PERT or a similar project analysis technique for all major government contracts. In 1959, Malcolm et al., described the computation of “slack” in the PERT system. “Slack” was the same concept as float in Critical Path Method (CPM), however slack was defined as the difference between the latest and the expected times at which an event will occur. In 1959, Kelley and Walker, held that the PERT model did not include a continuous probability distribution for the range of possible job durations, rather it included three values for minimum (optimistic), most likely and maximum (pessimistic) possible duration for each activity. In 1967, Schoderbek and Digman, developed routines associated with cost. PERT simulation had not become yet a cost-effective approach until this time. In 1969, Cantor and Dimsdale, gave with demonstration an exact algorithm which minimizes the number of nodes for Activity on Arrow (AOA) in a polynomial time. In 1971, King, has shown that calibration for PERT by historical data on the ratios between actual and estimated times is useful. PERT lacks calibration and has a strong tendency to underestimate stochastic variation (due to the independence assumption). The need to calibrate has been recognized right from the start by Malcolm et al. (1959). However, PERT remains uncelebrated to this day. In 1980, Krishnamoorthy and Deon, showed that finding the minimum dummy arcs problem is non deterministic polynomial time hard and they offered more heuristics. In 1990, Hayes gave a set of techniques to build a PERT network. In general, his method did not produce the minimal PERT network but he observed that the number of dummy arcs and nodes in the Activity on Arrow network could not be minimized simultaneously. Dimsdale proposed an algorithm’s construction of a minimal PERT network. Fisher et al. claimed that Dimsdale’s algorithm is false and gave a new and exact one but with no mathematical proof. In 2007 Cohen and Sadeh presented a new approach for drawing a unique PERT network that has a minimal number of dummy arcs. In 2013, Remon Fayek, proposed the development of simplified software called Repetitive-Projects Evaluation and Review Technique (RPERT), which is processed by Program Evaluation and Review Technique (PERT) integrated with Line Of Balance technique (LOB) for repetitive construction projects with identical activities in order to find the expected completion probability within a specified/certain duration (contract duration) using java programming code system. Up to this point it is reasonable to believe that there is a way to combine the two methods together.

REPETITIVE PROJECTS EVALUATION & REVIEW TECHNIQUE SOFTWARE DESCRIPTION

Microsoft EXCEL is used for the programming. As any project schedule the input data required is (1) The unit of the project (Day or Week); (2) The project end date. The user can choose to use the end date of the project as the schedule (in this case the late end date for the project will equal to the early end date calculated from the forward path) or enter another end date as mentioned in the contractor. If the end date entered less than the schedule end date there will be negative floats; (3) Activity table, which consists from the name of activities, predecessor activity, relationship between them and lag duration.; (4) For each entered activity three durations are required: Optimistic, Most likely and Pessimistic; (5) Time required calculating the probability of finishing the project in it; (6) And/or the probability desired for finishing the project. When the user finishes entering input data the program will start first to calculate duration for each activity that will be used in solving the network. It is known that the duration for any activity (O.D) depends on the quantity and the production rate for the labor or equipment.

$$O.D = \text{Quantity} / \text{Production Rate} \quad (1)$$

As PERT method the network must be solved using the estimated time (t_e) according to the three input time data for each activity and these three times are equivalent to three production rates: Optimistic (O), Most likely (M) and Pessimistic (P). The time estimated (t_e) is calculated as follows;

$$t_e = (O + 4M + P) / 6 \quad (2)$$

After calculating the duration for each activity, the program starts to calculate the dates for each activity of the project (early and late). Dates will be calculated as the same way in the Line of Balance method. One of the most important aspects of this method is the ability to specify one relationship type among different activities also, it maintain the resource continuity usage. To specify such relationships between two consecutive activities, the production rate of each activity is compared with that of its successors. If $O.D_i$ and r_i express unit duration and production rate of activity ‘i’, respectively, then:

$$r_i = 1/O.D_i \quad (3)$$

The activity under consideration will be referred to as current activity. If r_c and r_s express production rates of current and succeeding activities, respectively, the possible cases that may be encountered are (a) $r_s < r_c$: This implies that the start of the 1st unit of the succeeding activity is controlled by the finish of the 1st unit of the current activity; or (b) $r_s > r_c$: In this case, the start of the last unit of the succeeding activity is controlled by the finish of the last unit of the current activity. After specifying the relationship type between consecutive activities, the total duration (D) of an activity 'i' is calculated;

$$D_i = n * t_{ei} \quad (4)$$

Where; (D_i), is calculated as the sum of unit duration of all repetitive units, (n) is number of units. Having the relationship type among activities determined with their associated lags, network calculations similar to that of Precedence diagram method (PDM) are done. Forward path calculations are done to determine the early times of each activity, while the backward path determines the late times. Also, total floats are calculated for each activity and the critical activities are then specified. To calculate the probability of finishing the project in certain date Standard Normal Distribution tables are used. To use the tables it will be needed the value (Z);

$$Z = X_i - u / \partial_p \quad (5)$$

Where; (X_i) is the duration required to finish the project in; (u) is project total duration; (∂_p) is the standard deviation for the project. So in order to calculate this value the variance (∂) is calculated for each activity.

$$\partial = (P - O) / 6 \quad (7)$$

Then calculate the standard deviation for the whole project (∂_p);

$$\partial_p = (\partial_i + \partial_j + \partial_k \dots)^{0.5} \quad (8)$$

Where 'i', 'j', & 'k' are activities on the critical path. If there is more than critical path the program will calculate the standard deviation for each path and the maximum standard deviation will be the standard deviation for the whole project (∂_p). Then Table 1 and 2 are used to find out the probability.

After entering the input data, program starts to solve the schedule of the project, then the outputs are organized in tables as the following; (1) Early Start & Early Finish dates for first and last units; (2) Late Start & late Finish dates for first and last units; (3) Number of critical paths; (4) Total Float for each activity & determine the critical activities; (5) The probability of finishing the project in certain date; (6) The duration that satisfies the input probability of finishing the project; (7) Line of Balance charts for early dates & late dates. This program is limited by 20 relationships and the input data project end date must equal to or exceed calculated project end date to avoid negative floats. The program has been verified using a set of existing networks. All of the used networks have been solved manually to prove the validity of the program. The validation process is divided in two main stages. First validating the program linear solutions for a set of activities with regard to early and late dates, number of critical paths, total floats and critical activities. Then the probability of finishing the project in certain date is verified by solving the network for single unit using PERT manually.

EFFECT OF NUMBER OF REPETITION

There are several factors that affect the probability of finishing the project that repeats linearly. The probability of finishing the project (Z) depends on the standard deviation of the project (∂_p) which is calculated from the variance all every activities that lay on the critical path. The standard deviation of a project (∂_p) that consists from three activities A,B and C can be calculated from the following equation;

$$\partial_p = (\partial_A + \partial_B + \partial_C)^{0.5} \quad (9)$$

Where 'A', 'B', & 'C' are all on the critical path and (∂) is the variance for each activity, which is calculated from optimistic and pessimistic duration.

$$\partial = (P - O) / 6 \quad (10)$$

If this project is repeated linearly for two times then ($\hat{\sigma}_{p2}$) will be;

$$\hat{\sigma}_{p2} = (\hat{\sigma}_A + \hat{\sigma}_A + \hat{\sigma}_B + \hat{\sigma}_B + \hat{\sigma}_C + \hat{\sigma}_C)^{0.5} \tag{11}$$

While for three times then ($\hat{\sigma}_{p3}$) will be;

$$\hat{\sigma}_{p3} = (\hat{\sigma}_A + \hat{\sigma}_A + \hat{\sigma}_A + \hat{\sigma}_B + \hat{\sigma}_B + \hat{\sigma}_B + \hat{\sigma}_C + \hat{\sigma}_C + \hat{\sigma}_C)^{0.5} \tag{12}$$

Or the relation can be expressed as;

$$\hat{\sigma}_{p3} = (3^{0.5}) * (\hat{\sigma}_A + \hat{\sigma}_B + \hat{\sigma}_C)^{0.5} \tag{13}$$

And by using (n) as an indicator for number of repetition the equation will be;

$$\hat{\sigma}_{pn} = (n^{0.5}) * (\hat{\sigma}_A + \hat{\sigma}_B + \hat{\sigma}_C)^{0.5} \tag{14}$$

Where; ($\hat{\sigma}_{pn}$) is the Standard Deviation for the project that repeated (n) time; (n) is the Number of repetition; ($\hat{\sigma}_A, \hat{\sigma}_B, \hat{\sigma}_C$) are the Variances for the activities on the critical path. This equation determines the effect of number of repetition on the standard deviation of a linear project and therefore on the probability of finishing the project. If the activities on the critical path remain the same after repetition the equation can be simplified to the following;

$$\hat{\sigma}_{pn} = (n^{0.5}) * \hat{\sigma}_{p1} \tag{15}$$

To validate the program with the derived equation, a project "V" with the following data is used :-

Table 1 Activity Table For Project "V"

Activity	Optimistic Duration	Most Likely Duration	Pessimistic Duration	Depend On	Relation	Lag
A	10	12	14	-	-	-
B	8	10	12	A	FF	10
C	3	4	5	B	FF	4
D	7	7	13	B	FF	8
E	4	5	12	B	FF	6
F	10	11	12	B	SS	10
G	4	4	10	C	SS	4
				D	FF	5
				E	FF	5
				F	FF	5
H	5	5	5	G	SS	5
I	3	4	6	C	SS	4
				H	FF	4
J	6	8	10	H	SS	5
K	2	5	8	H	SS	5
L	8	10	12	I	SS	4
				J	SS	8

The project has been solved using the EXCEL program. The Time estimated and variances for each activity are listed in Table 2.

Table 2 Time Estimated And Variance for Activities In Project "V"

Activity	t_e	$\hat{\sigma}$
A	12	0.67
B	10	0.67
C	4	0.33
D	8	1.00
E	6	1.33

F	11	0.33
G	5	1.00
H	5	0.00
I	4	0.33
J	8	0.67
K	5	1.00
L	10	0.67

It can be noticed from output data that; (1) The numbers of critical paths are determined and it has only one critical path; (2) Critical Activities are (A, B, F, G, H, J & L). (3) Project duration is 61 days. The program used the estimated time (t_e) in the scheduling. The estimated time is calculated manually and the variance for each activity in order to find out the standard deviation ($\hat{\sigma}_p$) of project "V".

The standard deviation for the project ($\hat{\sigma}_p$) according to the critical path is equal to;

$$\hat{\sigma}_{p1} = (1^{0.5}) * (\hat{\sigma}_A + \hat{\sigma}_B + \hat{\sigma}_F + \hat{\sigma}_G + \hat{\sigma}_H + \hat{\sigma}_J + \hat{\sigma}_L)^{0.5} = 1.704$$

If this project is repeated linearly twice of times the duration of the project will be 79 days and the critical path will remain the same, so the standard deviation for the project ($\hat{\sigma}_p$) is equal to;

$$\hat{\sigma}_{p2} = (2^{0.5}) * (\hat{\sigma}_A + \hat{\sigma}_B + \hat{\sigma}_F + \hat{\sigma}_G + \hat{\sigma}_H + \hat{\sigma}_J + \hat{\sigma}_L)^{0.5} = 2.041$$

And by repeating the project three times the duration of the project is 97 days and the critical path still the same so the standard deviation for the project ($\hat{\sigma}_p$) is equal to;

$$\hat{\sigma}_{p3} = (3^{0.5}) * (2.9045)^{0.5} = 2.952$$

From the results, it is clear that the number of stories drastically affects the project variance, that for a great number of repetition, the probability of finishing the project at the desired end date will not be trustful.

EFFECT OF ACTIVITIES VARIANCE

The second goal from the study is to introduce the importance of studying the non-critical paths with low variances. In this case, if the project end date is required to be earlier for any circumstances, normally the critical path will have the lower probability to reach this date than the other paths, however in this case since the non-critical path has a higher variance than the critical one, it will have a lower probability than the critical path. In short, the critical path in this case will change from the original one to another path regarding the proximity of reaching the desired date. This is due to the inaccurate and shortage of information about the Original Duration. The standard deviation of a project ($\hat{\sigma}_p$) depends mainly on the variances of the activities, which are on the critical path, and if there is more than critical path the standard deviation is calculated twice and the bigger value is equal to the standard deviation of the project ($\hat{\sigma}_p$). However the variance of activities which are on the non critical paths may give a bigger standard deviation than the final standard deviation of the project ($\hat{\sigma}_p$) that is calculated and therefore it will affect the probability of finishing the project in the dates which is less than the critical one and that may cause major changes on the project. So even the non-critical path does not effect on project end date but it can effect on the probability of finishing the project in later dates. For example for a project "X" that repeated linearly 5 times where the project data are illustrated in Table 3;

Table 3 Activity Table For Project "X"

Activity	Optimistic Duration	Most Likely Duration	Pessimistic Duration	Depend On	Relation	Lag
A	11	12	13	-	-	-
B	9	10	11	A	FF	10
C	3	4	5	B	FF	4
D	7	8	9	B	FF	8
E	4	5	12	B	FF	6
F	5	5	35	B	SS	10
G	3	4	5	C	SS	4
				D	FF	5
				E	FF	5
				F	FF	5
H	4	5	6	G	SS	5
I	3	4	5	C	SS	4
				H	FF	4
J	4	4	28	H	SS	5

K	10	11	12	H	SS	5
L	3	3	45	I	SS	4
				J	SS	8

This project consists from 12 paths 2 of them only are critical and by solving this project using the EXCEL program it can be found that (1) Project end date is 133 days; (2) The standard deviation of the project ($\hat{\sigma}_p$) = 21.26; (3) The probability of ending the project in 129 days is 42.47%. It is clear from the pervious activity table that there are some activities have inaccurate data and/or problems in material supplies that may causes a great delay on them (Activity F, J & L) and this appear clearly by comparing the variance of activities;

Table 4 Time Estimated And Variance for Activities In Project "X"

Activity	t_e	$\hat{\sigma}$
A	12	0.33
B	10	0.33
C	4	0.33
D	8	0.33
E	6	1.33
F	10	5.00
G	4	0.33
H	5	0.33
I	4	0.33
J	8	4.00
K	11	0.33
L	10	7.00

By studying another path that is non-critical for example (A, B, D, G, H & K) it can be noticed that; (1) This path duration is 130 days and for that it is a non-critical path; (2) The probability of ending the project in 129 days is 29.12%. This show how the accurate information in the input data of the duration of each activity is very important in order to find out the exact probability of finishing any project in certain date and in case that the variance of some activities is un logic the data must be revised and if there is an acceptable reason for this data all paths of the project must be taken in consideration while calculating the probability of finishing the project in certain date. In this case the variances of activities are critical factors that affect the probability of finishing projects.

PROGRAM INPUTS AND OUTPUTS FOR SOLVED NETWORKS

The following screen shots illustrates the inputs and some of the outputs of the EXCEL program for project "X" which has been used in part 6.

Input red cells :

1.Unit =	Day
2.Project end date =	AS SCHEDULE
3.No. Of Repetition (n) =	5

4.Activity & Relationship Table:-

ACTIVITY	O.D.	DEPEND ON	RELATION	LAG
A	12.0	-	-	-
B	10.0	A	FF	10
C	4.0	B	FF	4
D	8.0	B	FF	8
E	6.0	B	FF	6
F	10.0	B	SS	10
G	4.0	C	SS	4
G	4.0	D	FF	5
G	4.0	E	FF	5
G	4.0	F	FF	5
H	5.0	G	SS	5
I	4.0	C	SS	4
I	4.0	H	FF	4
J	8.0	H	SS	5
K	11.0	H	SS	5
L	10.0	I	SS	4
L	10.0	J	SS	8

Figure 1 Screen shot 1 for Input Data

5. Activity Durations:-

ACTIVITY	OPTIMISTIC DURATION	MOST LIKELY DURATION	PESSIMISTIC DURATION
A	11	12	13
B	9	10	11
C	3	4	5
D	7	8	9
E	4	5	12
F	5	5	35
G	3	4	5
G	3	4	5
G	3	4	5
G	3	4	5
H	4	5	6
I	3	4	5
I	3	4	5
J	4	4	28
K	10	11	12
L	3	3	45
L	3	3	45

6. Probability of finishing project in = 129
 7. Project duration which satisfy a probability = 92.00%

Calculate

Figure 2 Screen shot 2 for Input Data

Output Data :

1. Early Dates:-

Activity name	START (1)	FINISH (1)	START (n)	FINISH (n)
A	0	12	48	60
B	20	30	60	70
C	54	58	70	74
D	38	46	70	78
E	46	52	70	76
F	30	40	70	80
G	65	69	81	85
H	70	75	90	95
I	79	83	95	99
J	75	83	107	115
K	75	86	119	130
L	83	93	123	133

Figure 3 Program Output For Project "X" - Early Dates

2.Late Dates:-

Activity name	START (1)	FINISH (1)	START (n)	FINISH (n)
A	0	12	48	60
B	20	30	60	70
C	61	65	77	81
D	40	48	72	80
E	50	56	74	80
F	30	40	70	80
G	65	69	81	85
H	70	75	90	95
I	79	83	95	99
J	75	83	107	115
K	78	89	122	133
L	83	93	123	133

Figure 4 Program Output For Project "X" - Late Dates

3.No of Critical Paths= 2

4.Floats & Critical Activity

Activity name	TF	Critical Activity
A	0.00	A
B	0.00	B
F	0.00	F
G	0.00	G
G	0.00	G
G	0.00	G
G	0.00	G
H	0.00	H
I	0.00	I
I	0.00	I
J	0.00	J
L	0.00	L

5.Probability of finishing project in = 129 is 42.47%

6.Project duration which satisfy a probability = 92.00% is 163

Figure 5 Program Output For Project "X" -Total Floats, Critical Activities & Probabilities

CONCLUSION

This research introduces an EXCEL program to solve repetitive construction projects using Line of Balance (LOB) method, applies PERT method on them, and find out the probability of finishing projects in certain dates using Normal Distribution Curves. This program seeks to confirm the following objectives: (1) Program Evaluation & Review Technique (PERT) is one of the most popular methods used for time estimating; (2) One of the most disadvantages of this method that it does not used to be applied on projects that consist of repetitive activities; (3) Probability of finishing projects depends mainly on the standard deviation of the project. (4) Number of repetition is a factor that affects the standard deviation of projects that repeat linearly. (5) Activities that have very big variance and long durations may give significant wrong probability of project finishing in certain time so using accurate information for activities duration guarantee more accurate time schedules. Further study can be done on: (1) Finding the probability of finishing single activity in the project in certain date; (2) Study the accurate affect of changing variance of activities that repeat linear on the standard deviation of the project.

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